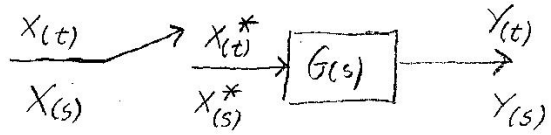
TRANSFORMADA DE LAPLACE ASTERISCO.

SEA:



ENTONCES:

$$Y_{(s)} = G_{(s)} \cdot X^*_{(s)}$$

MUESTREANDO AMBOS LADOS:

$$Y^*_{(s)} = \left[G_{(s)} \cdot X^*_{(s)}\right]^*$$

$$Y^*_{(s)} = G^*_{(s)} \cdot \left(X^*_{(s)}\right)^* = G^*_{(s)} \cdot X^*_{(s)}$$

ENTONCES:

$$Y_{(z)} = G_{(z)} \cdot X_{(z)} \Rightarrow$$

$$\frac{Y_{(z)}}{X_{(z)}} = G_{(z)}$$

FUNCIÓN DE TRANSFERENCIA PULSO DE DIAGRAMAS DE BLOQUES CON MUESTREADORES.

PROPIEDADES BASICAS:

1) $Y_{(s)} = G_{(s)} \cdot X^*_{(s)}$

$$Y^*_{(s)} = G^*_{(s)} \cdot X^*_{(s)} \Rightarrow Y_{(z)} = G_{(z)} \cdot X_{(z)}$$

2) $Y_{(s)} = G_{(s)} \cdot X_{(s)}$

$$Y^*_{(s)} = GX^*_{(s)}$$

$$Y_{(z)} = GX_{(z)} = \mathcal{Z}\left\{G_{(s)} \cdot X_{(s)}\right\}$$

EJEMPLO

DADO: $X_{(s)} = \frac{1}{s}$; $G_{(s)} = \frac{1}{s}$

OBTENGA:

a) $Y_{(z)} = G_{(z)} \cdot X_{(z)}$

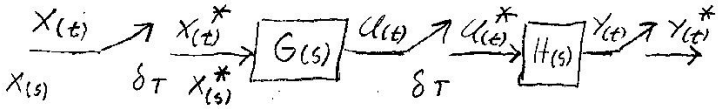
$$Y_{(z)} = \frac{1}{1-z^{-1}} \cdot \frac{1}{1-z^{-1}} = \frac{1}{(1-z^{-1})^2}$$

b) $Y_{(z)} = GX_{(z)} = \mathcal{Z}\left\{\frac{1}{s} \cdot \frac{1}{s}\right\}$

$$Y_{(z)} = \mathcal{Z}\left\{\frac{1}{s^2}\right\} = \frac{Tz^{-1}}{(1-z^{-1})^2}$$

EJEMPLO 1

OBTENGA LA FUNCIÓN DE TRANSFERENCIA PULSO DE:



Solución:

1) $U_{(s)} = G_{(s)} \cdot X^*_{(s)} \Rightarrow U^*_{(s)} = G^*_{(s)} \cdot X^*_{(s)}$

2) $Y_{(s)} = H_{(s)} \cdot U^*_{(s)}$

$$Y_{(s)} = H_{(s)} \cdot G^*_{(s)} \cdot X^*_{(s)}$$

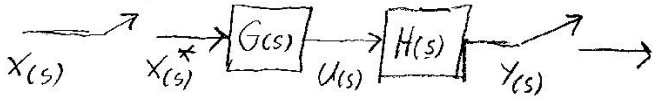
$$Y^*_{(s)} = H^*_{(s)} \cdot G^*_{(s)} \cdot X^*_{(s)}$$

$$Y_{(z)} = H_{(z)} \cdot G_{(z)} \cdot X_{(z)}$$

$$\frac{Y_{(z)}}{X_{(z)}} = H_{(z)} \cdot G_{(z)}$$

EJEMPLO 2

OBTENGA $\frac{Y(z)}{X(z)}$



Solución:

1) $U(s) = G(s)\, X(s)^*$ ①

2) $Y(s) = H(s) \cdot U(s)$ ②

SUST ① EN ②

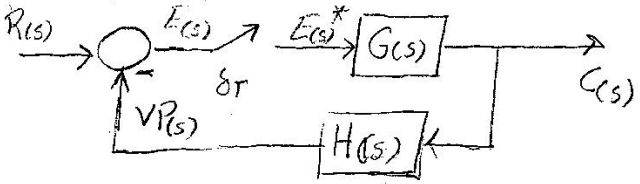
$Y(s) = H(s) \cdot G(s) \cdot X(s)^*$

$Y(s)^* = HG(s)^* \cdot X(s)^*$

$Y(z) = HG(z) \cdot X(z) \Rightarrow \frac{Y(z)}{X(z)} = GH(z)$

EJEMPLO 3

OBTENGA LA FUNCIÓN DE TRANSFERENCIA



ECUACIONES:

1) $C(s) = G(s) \cdot E(s)^*$

2) $VP(s)^* = H(s) \cdot C(s)$

3) $E(s) = R(s) - VP(s)$

• - PRIMER CAMINO:

3) $E(s)^* = R(s)^* - VP(s)^*$ SUST EN ①

$C(s) = G(s) \cdot R(s)^* - G(s) \cdot VP(s)^* \ldots$ Ⅰ

SUST ② EN Ⅰ

$C(s) = G(s) \cdot R(s)^* - G(s)\, H(s) \cdot C(s)$

$C(s)^* = G(s)^* \cdot R(s)^* - GHC(s)^*$ ← NO SE PUEDE SEPARAR.

• - SEGUNDO CAMINO:

SUST ① EN ②

$VP(s)^* = H(s)\, G(s) \cdot E(s)^* \ldots$ Ⅰ

APLICO ASTERISCO EN ②

$E(s)^* = R(s)^* - VP(s)^* \ldots$ Ⅱ

SUST Ⅰ EN Ⅱ

$E(s)^* = R(s)^* - H(s)G(s) \cdot E(s)^*$

$E(s)^* = R(s)^* - HG(s)^* \cdot E(s)^*$

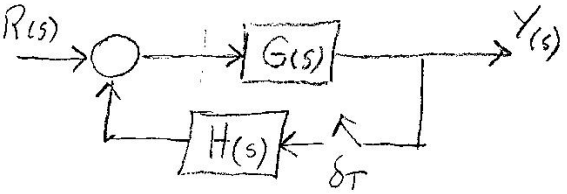
$E(s)^* = \frac{R(s)^*}{1 + GH(s)^*}$ SUST EN ①*

$C(s)^* = G(s)^* \cdot E(s)^*$

$C(s)^* = \frac{G(s)^* \cdot R(s)^*}{1 + GH(s)^*} \Rightarrow$

$$\boxed{\frac{C(z)}{R(z)} = \frac{G(z)}{1 + GH(z)}}$$

EJERCICIO:



$C(z) = \frac{GR(z)}{1 + GH(z)}$